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Thermodynamical Behaviour of Composite Stringy Black Holes

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ABSTRACT

We study the thermodynamical and geometrical behaviour of the black holes that arise as solutions of the heterotic string action. We discuss the near-horizon scaling behaviour of the solutions that are described by two-dimensional Anti-de Sitter space (AdS$_2$). We find that finite-energy excitations of AdS$_2$ are suppressed only for scaling limits characterized by a dilaton constant near the horizon, whereas this suppression does not occur when the dilaton is non-constant.

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1 Introduction

The charged black hole solutions of string theory in four dimensions [1, 2, 3] have been investigated for different purposes. From the point of view of General Relativity (GR) they generalize the Reissner-Nordstrom (RN) solutions of the Einstein-Maxwell theory. From the point of view of string theory they represent low-energy geometrical structures that should give some information about the fundamental string dynamics [4, 5].

More recently, these solutions have become interesting also from the point view of the Anti-de Sitter (AdS)/Conformal Field Theory (CFT) duality [6, 7]. In fact, it is well known that in the near-horizon, near-extremal regime the RN-like charged black hole solutions of string theory behave like $AdS_2 \times S^2$. They can be, therefore, very useful for trying to understand better the puzzles of the AdS_2/CFT_1 duality [8, 9]. In Ref. [10] a detailed study was performed about the arising of AdS_2 as near-horizon geometry of the RN solution of the Einstein-Maxwell theory. It was found that the finite-energy excitations of AdS_2 are suppressed. Only zero-energy configuration survive, whose degeneration should, in principle, be able to explain the entropy of the near-extremal RN black hole.

In this paper we extend the discussion of Ref. [10] to the black hole solutions in the context of heterotic string theory. Owing to the string/string /string triality of heterotic, type IIA and type IIB strings in four dimensions [11], our discussion also holds for the black hole solutions of type IIA and type IIB string theory. It is well known that the most general solution of this kind represents a generalization of the RN black hole. The new feature with respect to the RN case is represented by the presence of scalar fields (the dilaton and the moduli).

Using duality symmetry arguments we argue that the relevant information about the solutions is encoded in a single-scalar single-U(1)-field solution. We

analyze in detail both from the geometrical and thermodynamical point of view this black hole solution. We find that, whereas the geometrical structure of the the near-horizon solutions is the same as in the pure RN case, the presence of the dilaton allows both for solutions with constant dilaton and solutions with non-constant dilaton. Whereas in the former case finite-energy excitations of AdS_2 are still suppressed in the latter they are allowed.

The structure of the paper is the following. In Sect. 2 we show as the general black hole solution of heterotic string theory can be written as a single-scalar single-U(1) field solutions with RN causal structure. In Sect. 3 we discuss the geometrical and the thermodynamical behaviour of the solutions in the near-extremal, near-horizon limit.

2   Black hole solutions

The truncated version of the bosonic action for the heterotic string compactified on a six-torus is the following [12]

$$
\begin{aligned}
S \;=\; & \frac{1}{16\pi G} \int d^4x \sqrt{g} \Big\{ R - \frac{1}{2} \left[(\partial\eta)^2 + (\partial\tau)^2 + (\partial\rho)^2 \right] \\
& - \frac{e^{-\eta}}{4} \left[e^{-\tau-\rho} F_1^2 + e^{-\tau+\rho} F_2^2 + e^{\tau+\rho} F_3^2 + e^{\tau-\rho} F_4^2 \right] \Big\}.
\end{aligned}
\tag{1}
$$

In this action, we have set to zero the axion fields and all the $U(1)$ fields but four, two Kaluza-Klein fields F_1, F_2 and two winding modes F_3, F_4. The scalar fields are related to the standard definitions of the string coupling, Kähler form and complex structure of the torus,

$$
e^{-\eta} = ImS \qquad e^{-\tau} = ImT \qquad e^{-\rho} = ImU.
\tag{2}
$$

The most general, non-extremal (dyonic) solution in the Einstein-Hilbert frame is given by [13, 2]

$$
ds^2 \;=\; -(H_1 H_2 H_3 H_4)^{1/2} f dt^2 + (H_1 H_2 H_3 H_4)^{-1/2} \left(f^{-1} dr^2 + r^2 d\Omega_2^2 \right),
$$

$$e^{2\eta} = \frac{H_1 H_3}{H_2 H_4}, \quad e^{2\tau} = \frac{H_1 H_4}{H_2 H_3}, \quad e^{2\rho} = \frac{H_1 H_2}{H_3 H_4}$$

$$F_1 = dH_1 \wedge dt, \quad \tilde{F}_2 = dH_2 \wedge dt, \quad F_3 = dH_3 \wedge dt, \quad \tilde{F}_4 = dH_4 \wedge dt \quad (3)$$

where $\tilde{F}_2 = e^{-\eta-\tau+\rho*}F_2$, $\tilde{F}_4 = e^{-\eta+\tau-\rho*}F_4$ (* denotes the Hodge dual) and $H_i, f, i = 1 \ldots 4$, are given in terms of harmonic functions,

$$H_i = \left(g_i + \frac{\mu \sinh^2 \alpha_i}{r} \right)^{-1}, \quad f = 1 - \frac{\mu}{r}. \quad (4)$$

The extremal limit is obtained by

$$\mu \to 0 \qquad \sinh^2 \alpha_i \to \infty \qquad \mu \sinh^2 \alpha_i \to q_i. \quad (5)$$

For particular values of the parameters the solutions (3) can be put in correspondence with the solutions of the effective dilaton gravity action

$$S_{eff} = \frac{1}{16\pi G} \int d^4x \sqrt{g} \left[R - 2\left(\partial \Phi\right)^2 + e^{-2a\Phi} F^2 \right] \quad (6)$$

with a given by one of the following four values [2]

$$a = 0 \qquad a = \frac{1}{\sqrt{3}} \qquad a = 1 \qquad a = \sqrt{3} \quad (7)$$

The case $a = 0$ describes the Reissner-Nordström black hole of GR and corresponds to $H_1 = H_2 = H_3 = H_4$ in Eq. (3), whereas $a = \sqrt{3}, a = 1\, a = 1/\sqrt{3}$ correspond, respectively, to $H_2 = H_3 = H_4 = 1$, $H_1 = H_2$, $H_3 = H_4 = 1$, $H_1 = H_2 = H_3$, $H_4 = 1$.

A very interesting proposal is the so called *compositeness idea*, according to which the $a = \sqrt{3}$ solution can be seen as a fundamental state of which the other solutions are bound states with zero binding energy [4, 14, 15]. This idea stems basically from the higher dimensional interpretation of black holes as intersections of D-branes. The number of individual components is denoted by n. Elementary $n = 1$ solutions correspond to dilaton gravity theories with $a = \sqrt{3}$; $n = 2$ bound states correspond to $a = 1$ solutions,

$n = 3$ to $a = 1/\sqrt{3}$ and finally $n = 4$ correspond to $a = 0$. Moreover, the compositeness idea has been used together with the duality symmetries (in particular the $O(3, Z)$ duality group) of the model to generate the whole spectrum of BPS states [15].

The purpose of this paper is to study in detail the thermodynamical and geometrical behaviour of the stringy composite black holes (3) and to establish which relation they have towards Reissner-Nordström black holes.

The general solution (3) is very complicated to study. It contains nine arbitrary integration constants (four moduli g_i, four $U(1)$-charges and the mass); thanks to the $O(3, Z)$ duality symmetry of the model, all the relevant information about the nature of these black holes can be obtained by studying some simplified models which we get as we move in the moduli and in the charge space. Following the spirit of the compositeness idea, we can study those solutions we get if we equate some of the charges and of the moduli. In this way we will construct solutions with $\alpha_i \neq 0$ that describe single-scalar single-U(1)-field black holes and whose strong (or weak) coupling regime is exactly given by the dilaton gravity solutions of the model (6).

This can be done in a systematic way by exploiting the $O(3, Z)$ duality symmetry of the model in a way similar to that followed in Ref. [15] in dealing with the case of some null charges. The solutions can be characterized by giving the number m of equal moduli and charges (we consider only solutions with the same number of equal charges and moduli). It is evident that m is invariant under the action of the $O(3, Z)$ duality group described in Ref. [15]. It can be therefore used to label different representations of the duality group. Because $m = 1$ is equivalent to $m = 3$ we will have three multiplets on which the duality group $O(3, Z)$ will act by changing the scalar and the $U(1)$-field but leaving the geometry of the solution unchanged.

Hence, it will be enough to consider just one representative solution for each multiplet, the whole multiplet can be obtained acting on this solution

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with the $O(3, Z)$ group. Because $m = 4$ is nothing but the well-known Reissner-Nordstrom solution, in the following we will consider only $m = N = 1, 2, 3$. We will set $\alpha_1 = \alpha_2 = \alpha_4$, $g_1 = g_2 = g_4$ for $N = 1, 3$ and $\alpha_1 = \alpha_3$, $\alpha_2 = \alpha_4$, $g_1 = g_3$, $g_2 = g_4$ for $N = 2$. Requiring the solution to be asymptotically Minkowskian will impose an additional constraint on the moduli: $\prod_{i=1}^{4} g_i = 1$.

The solution can be written in a simple form by introducing the parameters λ_i

$$\lambda_i = \frac{\mu \sinh^2 \alpha_i}{g_i}, \tag{8}$$

the scalar charge σ and the parameters $r_\pm$, defined as follows:

$$
\begin{aligned}
r_- &= \lambda_1, & r_+ = \mu + \lambda_1, & 2\sigma L_P = \lambda_3 - \lambda_1, & \text{for } N = 1, \\
r_- &= \lambda_1, & r_+ = \mu + \lambda_1, & 2\sigma L_P = \lambda_2 - \lambda_1, & \text{for } N = 2, \\
r_- &= \lambda_3, & r_+ = \mu + \lambda_3, & 2\sigma L_P = \lambda_1 - \lambda_3, & \text{for } N = 3, \tag{9}
\end{aligned}
$$

where L_P is the Planck length. The $U(1)$-charges q_i can be written in terms of the other parameters as follows (no summation on i)

$$q_i^2 = g_i^2 \lambda_i (\lambda_i + \mu) \tag{10}$$

With the previous positions and performing the coordinate change $r \rightarrow r - r_-$ the solution (3) becomes:

$$
\begin{aligned}
ds^2 &= -\frac{(r - r_+)(r - r_-)}{r^{(4-N)/2}(r + 2\sigma L_P)^{N/2}} dt^2 + \frac{r^{(4-N)/2}(r + 2\sigma L_P)^{N/2}}{(r - r_+)(r - r_-)} dr^2 \\
&+ r^{(4-N)/2}(r + 2\sigma L_P)^{N/2} d\Omega_2^2 \\
e^\eta &= e^{\eta_0} \left(1 + \frac{2\sigma L_P}{r}\right)^\gamma, \tag{11}
\end{aligned}
$$

where $\gamma = 1$ for $N = 2$ and $\gamma = \pm 1/2$ for $N = 1, 3$ respectively. For $N = 1, 3$ we also have $\tau = \rho = -\eta$ while for $N = 2$ we have $\tau = \rho = 0$ as can

be easily seen from eq. (3). The duality group $O(3, Z)$ acting on the solutions (11) with a given N generates the corresponding multiplet. The duality acts on the scalars whereas the metric part of the solution remains unchanged. For instance, the τ_S duality [15] ($\eta \rightarrow -\eta$, $F_1 \rightarrow \tilde{F}_3$, $F_3 \rightarrow \tilde{F}_1$, $F_2 \rightarrow \tilde{F}_4$, $F_4 \rightarrow \tilde{F}_2$), acting on the solution $N = 2$, exchanges the electric solutions with the magnetic ones.

The parameters r_+, r_-, σ are related to the mass M and U(1)-charges q_i by the following equations

$$L_P^2 Q^2 = r_+ r_-, \quad 2ML_P^2 = r_+ + r_- + N\sigma L_P, \tag{12}$$

where we have introduced the adimensional charge Q defined as

$$Q \equiv \frac{q_i}{L_P g_i} \tag{13}$$

where $i = 1$ for $N = 1, 2$ and $i = 3$ for $N = 3$. From eq. (12) follows the relation

$$r_\pm = L_P \left(L_P M - \frac{N}{2}\sigma \pm \sqrt{(L_P M - \frac{N}{2}\sigma)^2 - Q^2} \right) \tag{14}$$

as well as the extremality condition

$$L_P M \geq Q + \frac{N}{2}\sigma. \tag{15}$$

The solutions (11) represent a four-parameters generalization of the RN solution of GR. In the heterotic string context, the RN solution is described by the $m = 4$ multiplet, and it is obtained by putting $N = 0$ in Eq. (11).

One can easily show that the causal structure of the solutions (11) is the same as that of the RN solutions, the effect of the scalar charge σ being a shift of the $r = 0$ singularity. Solutions of this kind have been already discussed in the literature [16].

It is interesting to notice that the bound-state solutions with $n = 1, 2, 3$ elementary constituents can be obtained as strong (or weak for the dual

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solutions) coupling regime of the corresponding $m = 1, 2, 3$ solution. In fact, the former solutions are characterized by some vanishing $U(1)$-charges, whereas from Eq. (8) and (5) it follows that $q = 0$ can be achieved by $g \to \infty$. This fact has a natural explanation if one uses the bound state interpretation of the black holes. Since the bound state has zero binding energy, the mass of the composite state is the sum of the masses M_i of the elementary constituents. But M_i behaves as q_i/g_i so that in the strong coupling regime some of the elementary constituents of the $n = 4$ solution do not contribute, leaving an effective $n = 1, 2, 3$ bound state.

3 Thermodynamical behaviour

An important property of the RN black holes is that the semiclassical analysis of their thermodynamical behaviour breaks down very near extremality [17]: the formulae for the entropy and temperature are given by

$$
\begin{aligned}
S_{BH} &= \frac{\pi r_+^2}{L_P^2} \\
T_H &= \frac{1}{4\pi} \left(\frac{\partial g_{00}}{\partial r} \right)_{r=r_+} = \frac{r_+ - r_-}{4\pi r_+^2}.
\end{aligned}
\tag{16}
$$

Near extremality, the excitation energy above extremality E and temperature T_H are related in the following way

$$
E \equiv 2\pi^2 Q^3 T_H^2 L_P.
\tag{17}
$$

At an excitation energy of

$$
E_{gap} = \frac{1}{Q^3 L_P}
\tag{18}
$$

the semiclassical analysis ceases to be valid. The nature of this breakdown is well understood in string theory: the black hole develops a mass gap [18] and (18) is the energy of its lowest-lying excitation. Maldacena, Michelson

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and Strominger [10] studied both the geometry and the thermodynamical behaviour of the near-extremal RN black holes in the near-horizon limit. They showed that the spacetime always factorizes as $AdS_2 \times S^2$, with AdS_2 endowed with the Robinson-Bertotti metric. Moreover, they found that in this limit all the finite-energy excitations of AdS_2 are suppressed.

The previous features hold for the RN solution. One would like to know if they represent a general feature of the solutions with $AdS_2 \times S^2$ near-horizon geometry. Let us therefore consider the near-extremal, near-horizon thermodynamical behaviour of the black hole solutions (11).

The Hawking temperature of these solutions is given by

$$T_H = \frac{r_+ - r_-}{4\pi r_+^2} \left(\frac{r_+ + 2\sigma L_P}{r_+} \right)^{-N/2}, \tag{19}$$

whereas for the entropy we have,

$$S = \frac{\pi r_+^2}{L_P^2} \left(\frac{r_+ + 2\sigma L_P}{r_+} \right)^{N/2}. \tag{20}$$

As we are going to see see in detail soon, in the near-extremal, near-horizon regime the solutions (11) always behave as $AdS_2 \times S^2$. In this situation the energy-temperature relation is

$$E \equiv 2\pi^2 T_H^2 L_P Q^3 \left(\frac{Q + 2\sigma}{Q} \right)^N \tag{21}$$

and the energy (18) at which the semiclassical analysis breaks down is given by

$$E_{gap} = \frac{1}{Q^3 L_P} \left(\frac{Q}{Q + 2\sigma} \right)^N. \tag{22}$$

Eqs. (21) and (22) represent a generalization to the heterotic string of the formulae (17) and (18) of the pure RN case. The only difference between the two cases is the presence of the scalar charge σ, which is a consequence of the presence of a non-trivial dilaton. The near-horizon limit can be achieved as

in Ref.[10] by letting $L_P \to 0$ but holding fixed some of the remaining parameters E, T_H, Q, σ. In principle one could also consider more general limits for which T_H is not fixed but, as shown in Ref. [10], these cases present intricate features, in particular the geometry becomes singular. We performed a detailed analysis of the various limits for the three cases ($N = 1, 2, 3$) and found out that for finite σ the results obtained by Maldacena and others [10] for the RN black hole hold true also for these other solutions: the near-horizon geometry is always $AdS_2 \times S^2$ and there are no finite-energy excitations, despite the presence of the scalar charge. This result was somehow expected but never proved explicitly in literature.

The new feature appears when we take the limit $L_P \to 0$ together with $\sigma \to \infty$, holding E, T_H and Q fixed. In this case the near-horizon geometry is again $AdS_2 \times S^2$ and finite energy excitation of AdS_2 are allowed holding $L_P \sigma^N$ fixed. However in this case AdS_2 is endowed with a non-constant, linearly varying, dilaton. The resulting two-dimensional model is similar to that corresponding to the $a = 1/\sqrt{3}$ case in Eq. (6), which has been shown to be also relevant for the AdS_2/CFT_1 correspondence [9, 19]. We do not discuss here the limit $L_p \to 0, Q \to \infty, (T_H, \sigma)$ fixed because there is nothing new with respect to the pure RN case discussed in Ref. [10]. The relevant limits to be discussed are: (a) $L_p \to 0, (T_H, Q, \sigma)$ fixed; (b) $L_p \to 0, \sigma \to \infty$, (T_H, Q) fixed. In both cases the near-horizon geometry is $AdS_2 \times S^2$ but whereas in case (a) the dilaton is constant near the horizon in case (b) we have a non-constant, linearly varying dilaton.

(a) $L_p \to 0, (T_H, Q, \sigma)$ fixed

Defining

$$U = \frac{r - r_+}{L_P^2},\qquad(23)$$

and performing the limit in Eq. (11) keeping U fixed, we get

$$\frac{ds^2}{Lp^2} = -\frac{U^2 + 4\pi R^2 T_H U}{R^2} dt^2 + \frac{R^2}{U^2 + 4\pi R^2 T_H U} dU^2 + R^2 d\Omega_2^2,$$

$$e^\eta = e^{\eta_0} \left(\frac{R}{Q} \right)^{4\gamma/N}, \tag{24}$$

where R is given in terms of the two charges σ, Q:

$$R = Q \left(1 + \frac{2\sigma}{Q} \right)^{N/4}, \tag{25}$$

and γ is given as in Eq. (11). Hence, in this case the near-horizon geometry is the same as in the pure RN case, the dilaton being a constant near the horizon. What changes is just the radius R of the transverse two-sphere. In our case it is a function of both the $U(1)$- and scalar charges. From Eqs. (21) and (22) we obtain that in this limit the excitation energy goes to zero, whereas E_{gap} diverges. Analogously to the pure RN case we cannot have finite energy excitations of AdS_2.

(b) $L_p \to 0$, $\sigma \to \infty$, (T_H, Q, E) fixed

From Eqs. (21) and (22) it is evident that we can hold (T_H, Q, E) fixed while $L_p \to 0$ if we allow $\sigma \to \infty$, with $L_p \sigma^N \equiv const$. Let us define

$$V = \left(\frac{E_{gap}}{L_P^3 Q} \right)^{1/2} (r - r_+). \tag{26}$$

Performing the limit while keeping V fixed, the solution (11) becomes,

$$\frac{ds^2 E_{gap}}{Lp} = -(V^2 + 4\pi T_H V)dt^2 + \frac{1}{V^2 + 4\pi T_H V}dV^2 + d\Omega_2^2, \quad \eta \propto \ln V. \tag{27}$$

As in the previous case the near-horizon geometry is $AdS_2 \times S^2$, but now the dilaton is not constant. Thus, heterotic string black holes allow for finite-energy excitations of AdS_2, but they require a non constant dilaton. This feature has been also noticed in Ref. [10] in the analysis of the pure RN case.

It is interesting to notice that AdS_2 with a nonconstant dilaton has already emerged as the near-horizon geometry of the $n = 3$ ($a = 1/\sqrt{3}$ in Eq. (6)) heterotic string black hole [19]. AdS_2 endowed with a nonconstant

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dilaton has been also used to give a realization of the AdS_2/CFT_1 correspondence. Here we have shown that this kind of models can emerge also as near-horizon limit of RN-like four dimensional geometries. Moreover, the fact that in this context finite energy excitations of AdS_2 are allowed could be useful to circumvent some of the problems of the pure RN case.

Until now we have considered only solutions with all the four $U(1)$-charges different from zero. Let us now consider the solutions we obtain when some of the charges go to zero, i.e the composite black hole solutions with $n = 1, 2, 3$. Although the thermodynamics of these black holes has been already discussed in the literature [20], it is interesting to compare it with the thermodynamical behaviour of the $N = 1, 2, 3$ solutions.

The solutions describing bound states of $n = 1, 2, 3$ elementary black holes can be obtained as the $g \to \infty$, strong coupling regime, of the solution with $N = 1, 2, 3$. Because of Eq. (8) the corresponding solutions can be obtained putting $r_- = 0$ in Eq.(11). The inner horizon disappears and the causal structure of the solutions becomes radically different from that of the RN-like black holes. In the extremal limit the horizon matches the singularity which is timelike for $n = 1$ and null for $n = 2, 3$.

The Hawking temperature associated with the black hole is

$$T = \frac{1}{4\pi} \left(\frac{\partial g_{00}}{\partial r} \right)_{r=r_+} = \frac{1}{4\pi} (r_+)^{\frac{n}{2}-1} (r_+ + 2\sigma L_P)^{-n/2}. \tag{28}$$

In the near-extremal limit $r_+ = 2EL_P^2$ and thus we have the following energy-temperature relations

$$\begin{aligned} E &\sim \left(64\pi^2 L_P^3 \sigma T^2 \right)^{-1}, && \text{for} \quad n = 1, \\ T &\sim \frac{1}{8\pi\sigma L_P}, && \text{for} \quad n = 2, \\ E &\sim 64\pi^2 T^2 L_P \sigma^3, && \text{for} \quad n = 3. \end{aligned} \tag{29}$$

For $n = 1$ the specific heat is negative and this is probably related with the nature of the singularity, which in the extremal limit is timelike. For $n = 2$

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there is no dependence of the excitation energy on the temperature. This behaviour can be explained, at least in principle, in terms of the underlying two-dimensional model [19, 21]. Finally, for $n = 3$, the energy-temperature relation is similar to that of Eq. (17). This relation indicates that model has a sensible description in terms of an effective two-dimensional model that admits AdS_2 as solution [19]. The main difference with the RN-like case is that here the dilaton is not constant near the horizon.

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